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Roles of a teacher and researcher during in situ professional development around the implementation of mathematical modeling tasks

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Abstract Partnership with teachers for professional development has been considered beneficial because of the potential of collaborative work in the teacher's own classroom to be relevant to practice. From this perspective, both teachers and researchers can draw on their own expertise and work as authentic partners. In this study, we address the need for such collaboration and focus on how a teacher and a researcher performed their roles when collaboratively implementing mathematical modeling tasks within a context of in situ professional development. Using multi-tier design-based research, as a framework, a researcher worked in a teacher's classroom to implement a series of research-based mathematical modeling activities. A broad corpus of data from this interaction was analyzed, including audio recordings of interviews with the teacher, video recordings of three mathematical modeling lessons, researcher field notes and journal reflections, instructional materials, and students' written work using the principles for designing activities for teachers. The emerging roles and relationships between the teacher and the researcher were documented, as (1) the researcher implemented the professional development, (2) the teacher shared her concerns, (3) the researcher responded to the teacher's challenges, and (3) the teacher reflected on student development. As a case study of collaboration, the participants' roles and strategies to overcome challenges and achieve shared objectives can benefit teachers and researchers who plan to collaboratively implement modeling in the classroom. The study supports the value and viability of this form of in situ professional development, indicating that significant changes in teachers' thinking about their students' mathematical model development can occur in relatively short periods of time.

Keywords Teacher learning · Mathematical modeling · Professional learning community · Professional development

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Introduction

Several recent research studies have demonstrated the importance and value of an integrative approach to classroom-based education research (e.g., Ball and Cohen 1999; Darling-Hammond et al. 2009; Doerr and Lesh 2003; Hiebert et al. 2002). In this approach, teachers become active learners and primary decision makers, while researchers acknowledge the potential value of teachers' knowledge (Hiebert et al. 2002; Mundry et al. 2000). Both communities work together as partners, learning from each other (Hiebert et al. 2002). In contrast, in more traditional research settings, teachers are less likely to become decision makers because researchers have greater access to resources and maintain decision making (Goos 2008). The goal of an integrative approach in research is to have a more balanced relationship between teachers and researchers where teachers become active decision makers.

Through integrative research work in the context of modeling tasks, researchers in the models and modeling perspective (MMP) have developed *principles for designing activities for teachers* (e.g., Doerr and Lesh 2003), in which teachers have key active roles: interpreting their own students' work, addressing the diversity of student thinking they encounter, and iteratively evaluating their own teaching goals and assessment strategies. Because the modeling processes of interest to such research are happening at multiple levels (student-level modeling of real-life situations; teacher-level modeling of students' thinking; and researcher-level modeling of teachers' activity and decision making), this perspective is known as multi-tier design-based research, or DBR (Brown 1992; Cobb et al. 2003; The Design-Based Research Collective 2003). An important practical feature of this approach is that it does not take teachers away from the scene of their classroom to build new ideas about teaching mathematics; rather, their interpretation of their own students' work becomes the basis for the continuous development of their teaching (Schorr and Lesh 2003). As teachers interpret their students' thinking, researchers investigate how teachers use their own conceptual systems to understand students' thinking and make instructional decisions in the complex context of schools (Lesh and Kelly 2000).

In this study, we follow the integrative approach of multi-tier DBR, focusing on describing the relationship between a teacher (referred to with the pseudonym "Kate"), and a researcher (the first author) as this pair worked to implement a series of research-based mathematical modeling tasks for use in Kate's classroom. The study used model-eliciting activities or MEAs (Lesh et al. 2000; Lesh et al. 1992), which are particularly suited to multi-tier DBR because of their known potential for eliciting students' thinking and behaviors as they construct, share, evaluate, and modify their generalizable conceptual systems (Doerr and English 2006). These processes produce classroom discourse and artifacts that provide concrete opportunities for researchers and teachers to reflect together on students' diverse ways of thinking. Our study investigates how Kate used this experience as an opportunity for professional development, along with a researcher's roles as she helped Kate arrive at new perspectives on the nature and growth of her students' modeling perspectives. Specifically, we aim to answer the question: "How did the teacher and researcher establish roles and construct a relationship conducive to two-way, collaborative learning while implementing mathematical modeling tasks within a context of in situ professional development?"

Literature review

In this section, we summarize conceptions and processes of mathematical modeling as described in a policy document and in prior research studies. The principles of MEAs as modeling activities are then described to provide concrete examples of modeling processes. Finally, we introduce integrative research methods and a framework for teacher development that are aligned with the aim of this study.

Conceptions of models and modeling

Since 2010, 42 states have adopted a common set of K-12 mathematics standards, the *Common Core State Standards for Mathematics* (*CCSSM*) (National Governor's Association Center for Best Practices [NGA] and Council of Chief State School Officers [CCSSO] 2010), with the resulting expectation that a majority of K-12 teachers in the USA will begin teaching mathematics based on these common standards (Reys et al. 2012). *CCSSM* includes standards for mathematical practice that teachers are tasked with cultivating in their students at all levels across K-12 education. One of these practices is "Model with mathematics," which recommends that students "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA and CCSSO 2010, p. 7).

Lesh et al. (2013) defined modeling as "the process of developing a purposeful mathematical description (or interpretation) of a problem-solving or decision-making situation. Such processes often involve quantifying, dimensionalizing, coordinatizing, or (in general) mathematizing objects, relations, operations, patterns, and regularities which do not occur in pre-mathematized forms" (see also Lesh et al. 2007, p. 346). Mathematical modeling thus involves conceptual systems including elements, relationships among elements, operations describing the interaction among the elements, and patterns or rules (e.g., Doerr and English 2006; Lesh and Lehrer 2003). To provide concrete examples of modeling processes, we reviewed related studies describing model-eliciting activities (MEAs), which are designed to support learners in mathematizing reality.

Model-eliciting activities (MEAs)

An MEA is an activity wherein teams of students solve authentic, real-life mathematical tasks over relatively short time periods (one or two class sessions), describing, testing, evaluating, and revising their *models* (i.e., representational descriptions of the problem situation) (Lesh et al. 2000). The problem contexts of MEAs require students to interpret and respond to realistic situations, developing model-rich responses as the basis of their solutions. As students refine their approaches to the problem through iterative *modeling cycles*, they not only apply mathematical concepts that they have previously learned; they also develop new mathematical concepts. Principles for designing MEAs have been articulated in several past studies (Doerr and English 2006; Lesh et al. 2000; Lesh et al. 1992) (see Table 1).

Table 1 Principles for developing MEAs

Principles	Descriptions
The reality principle	Students interpret the task based on their own real-life experiences
The model construction principle	Students construct a conceptual system as they explain, extend, predict, or modify the model
The self-evaluation principle	Students judge their responses for themselves based on the statement of the problem including the criteria
The model generalizability principle	Students develop generalizable knowledge through the task that can be used in other situations
The model documentation principle	Students express the givens, goals, and possible solution paths
The simplest prototype principle	Students develop a mathematical model to the situation that is simple for students to remember

Relations between researchers and practitioners

In addition to creating a new genre of learning tasks that support both microgenetic and developmental research into learning and the growth of ideas, MMP researchers have worked to establish particular research methods that connect research with practice. In the beginning of the twentieth century, Thorndike used a new educational research method borrowed from the physical sciences, which emphasized "objectivity" in measurement, isolated variables, and quantitative results. His approach was accepted by many research-oriented universities and became a standard method for educational research (Hiebert et al. 2002). However, this method was later brought into question in part because it fostered an "authoritarian, manipulative, bureaucratic system" (Cazden 1983, p. 33), where researchers unfamiliar with the classroom environment often decided what changes teachers should make. This approach has also been criticized because the knowledge produced by such research tended to be abstract and isolated from the teachers' experiences and from classroom contexts (Hiebert et al. 2002), reducing its utility to inform or illuminate practice.

In contrast with Thorndike, Dewey and his colleagues, and their intellectual successors, took a more integrative approach focusing on collaborative work in teacher's classrooms, such as inquiry groups among teachers (Ball and Cohen 1999). Through this approach, the professional development (PD) experiences are directly related to the teachers' practice because teachers use resources to develop and reflect on their instructional decisions in their classrooms (Darling-Hammond and Ball 1998; Purnam and Borko 2000). Additionally, through PD experiences, teachers have opportunities to share their lesson planning and instruction with others and to engage in a community of practice (Darling-Hammond and McLaughlin 1995; Guskey 1995; Loucks-Horsley et al. 1987).

There is also a considerable international literature that focuses on professional learning communities; for example, one of the strands in the International Commission on Mathematics Instruction (ICMI) Study 15 was "professional learning in and for practice," which invited about 150 researchers from over 30 countries to discuss this theme (Silver et al. 2007). Lesson study is another form of professional community that has emerged among Japanese teachers over the past several years to collectively improve their mathematics and science instruction (Lewis 2002; Stigler and Hiebert 1999; Yoshida 1999). Additionally, in China, a groups of teachers collaboratively plan lessons where they explain lessons to others as a way to improve classroom practice (Peng 2007). Yang and

Ricks (2013) also described how groups of mathematics teachers in China improved their instructional strategies through Teaching Research activity.

In spite of this research literature, the USA generally lacks the kind of professional communities that encourage teachers to collaboratively make decisions and learn from one another through co-teaching and peer-feedback (Darling-Hammond et al. 2009). More collaborative work in teacher's classrooms is needed, where both researchers and teachers reorient their goals to be more collaborative and to incorporate their own expertise (Hiebert et al. 2002). According to this philosophy, teachers should be active learners during research and should be "primary judges" on matters of implementation and pedagogy, not "passive recipients" of massive streams of external information (Mundry et al. 2000). Researchers need to acknowledge the value and potential of the teachers' personal knowledge as professional knowledge, rather than undervaluing the knowledge and insights that teachers have gained from work in their own classrooms. Both communities can work together as authentic partners to gain from the other's knowledge (Hiebert et al. 2002).

Doerr and Lesh (2003) proposed that teaching is a complex process; teachers' knowledge is not a single or uniform quantity, but an evolving one. Therefore, PD for teachers should not be based on a "pre-determined standard of excellence" (p. 127), but must be constructed collaboratively during the research and implementation process itself. Also, teachers' professional learning experiences need to be related to and contextualized in their practice in order for the theory to be applied in the complex teaching settings (Ball and Bass 2003; Little 1993). This integrative method, focusing on collaborative work between teachers and researchers, is the approach that we utilized for this study.

Adapting the principles for designing MEAs to the context of teacher education

Multi-tier DBR emphasizes structural parallels between the various layers of modeling involved in the research. Thus, Doerr and Lesh (2003) developed principles intended to elicit teachers' models (e.g., teaching tools that reveal teachers' understanding of student learning) (as shown in Table 2) based on the six principles for designing MEAs in Table 1. Just as multi-tier DBR acknowledges structural parallels between its levels, so too are there connections between the six principles for MEA design and these principles for teacher-level modeling. In particular, the Sharing, Self-Evaluation, and Reality principle in Table 2

Table 2 Principles for designing activities for teachers

Principles	Descriptions
Multiple contexts principle	Activities for teachers address variability in the classroom, such as diversity of students, mathematical contexts of teaching, and classroom environment
Multilevel principle	Activities for teachers address the multiple aspects of teacher development, such as mathematical content and pedagogy
Sharing principle	Activities for teachers encourage them to share their ideas for teaching and learning with other teachers. Their tasks can be modified to be used by multiple teachers
Self-evaluation principle	Activities for teachers help them evaluate their own teaching goals and assessment strategies
Reality principle	Activities for teachers assist them with interpreting student work from their own classrooms or developing an assessment task that they use in their own practice

map to the Generalizability, Self-Evaluation, and Reality principle for student MEAs (Table 1), respectively. (There are also relations and analogies among the other principles, but the parallelism is less direct).

Teachers' models are often implicit and not discussed with colleagues. A key goal of activities that follow the principles shown in Table 2 is to support teachers in explicitly expressing, testing, and modifying those ways of thinking and sharing the interpretation system with others who can apply them to their own instruction (Doerr and Lesh 2003). Again, design research that applies these principles is multi-tiered because, while students engage in MEAs, teachers analyze their students' data, and develop their own models of teaching tools that show their understanding of student learning; and researchers review and reflect on the teachers' analyses of students' thinking, developing models of PD that express their evolving understanding of teachers' and students' learning (Lesh and Kelly 2000).

Methods

Within the theoretical framework of integrative multi-tiered DBR (Lesh and Kelly 2000), the teacher acts as an investigator (and participant), while the researcher also acts as a teacher/learner (and investigator). Adopting this framework, Hyunyi engaged in an 11-week partnership (about 100 h) with two experienced eighth-grade mathematics teachers at a middle school in a medium-sized Midwestern city in the USA. This school serves a fairly diverse, high-need student population. Approximately 70 % of students in this school receive the free or reduced lunch (mentioned in the interest of providing a rough

Table 3 Setting of the in situ PD and collected data

Week/day	Practice	Researcher	Teacher	Collected data
3/Mon	Task instruction	Introduced modeling tasks	Solved the tasks and chose one that was appropriate for her class	Audio recording of interaction between researcher and teacher
3/Mon	Co-planning	Provided discussion prompts and feedback	Planned the lesson and developed an observation list	Audio recording of planning, the observation list
3/T–F	Preparation	Analyzed audio recordings and prepared for teaching	Prepared for teaching	Researcher's journal reflection, modified modeling task
4/Mon	Co-teaching	Co-taught the lesson or observed teachers' instruction	Co-taught the lesson or observed researcher's instruction	Researcher's filed notes, video recordings of each lesson, students' written work
4/Mon	Debriefing	Provided discussion prompts and feedback	Reflected on teaching and students' learning	Audio recordings of debriefing
4/T–F	Analysis of learning	Analyzed audio and video recordings, and students' written work	Analyzed students' written work and developed a follow-up task	Researcher's journal reflection, the follow-up task

indication of the socioeconomic status of the school). The student body includes 12 % African Americans and 20 % Hispanic students. The teachers that Hyunyi engaged with taught mathematics in three full inclusion and two general education classes. (More advanced classes were taught by other teachers at the school.)

In this study, we describe Hyunyi and Kate's experiences, focusing on the changes in Kate's perspectives on her students' work over the extended implementation period. Kate's classroom was selected for particular attention because of her experience in special education and because of her classroom norms that were not initially conducive to the student-directed problem-solving work involved in MEAs. For example, it was common for Kate to introduce topics by demonstrating procedural approaches and asking her students to imitate these procedures when working similar problems. Thus, Kate was positioned for in situ PD with a rich resource of professional knowledge (her expertise in special education) on the one hand, and with an important learning challenge (the need to adapt to new forms of student interaction and new classroom task structures) on the other. In engaging with Kate, Hyunyi utilized six practices that are aligned with multi-tiered DBR. These included *Task Introduction, Co-planning, Preparation, Co-teaching, Debriefing*, and *Analysis of Student Learning*. Table 3 shows the setting of the in situ PD and collected data from each practice.

Settings and data analysis during the implementation

This section provides a brief description of the study setting, including data collection and analysis methods performed during the in situ PD. Hyunyi's objectives for her first and second visits included becoming familiar with Kate and her students and observing her pedagogical strategies and routines. As Kate engaged in her usual teaching practices, Hyunyi observed how Kate planned and taught her lessons, and she helped Kate with grading the students' assignments and quizzes. This process helped Hyunyi understand Kate's classroom, in general, including how Kate taught and what students learned in her classes. During the third visit, Hyunyi asked questions guided by a semi-structured interview protocol, including a core set of questions (e.g., What are your teaching goals with respect to student learning? What opportunities have students in your class had to collaborate or present their work in class? Which specific problems or activities have you used to teach mathematical modeling?). These questions were asked again for the eleventh visit and for a follow-up member-checking visit (after about a year from the first visit), in connection with the preparation of this manuscript.

After the first interview (week 3), Hyunyi introduced ideas about the processes of modeling and described MEAs related to the topics that the teachers were planning to teach (*Task Instruction* in Table 3). After working through these modeling tasks together, the teachers and Hyunyi discussed how they planned to implement a task (*Co-planning*). They then individually developed a list of items to observe when students worked in groups, and compared and discussed their lists. After co-planning, Hyunyi wrote journal reflections based on the analysis of audio recordings of her discussions with the teachers. The teachers and Hyunyi prepared for co-teaching, making revised instructional materials and PowerPoint slides to introduce students the context of the first task (the *Preparation* practice in Table 3).

During the fourth visit, the teachers and Hyunyi co-taught a modeling task and used their co-constructed guide to support their observations and interactions with students during the lesson (*Co-teaching*). Two video cameras were set up in each teacher's classroom for three classes (one for capturing whole-class interactions, the other focused on a

Table 4 Description of three model-eliciting activities (MEA)

MEA title	Description
MEA1: Summer jobs	List assumptions students make about what types of summer jobs (e.g., washing cars, lawn mowing, paper routes) Jack would do and estimate what his earnings might be
MEA2: Big foot	Write a letter to Sherlock Holmes informing him how he can use footprints to make good guesses about the height of the person who made them
MEA3: Volleyball	Write a letter to the organizers of a volleyball camp describing a procedure for developing a list of fair team members. A variety of information is provided, including individual's height, vertical leap, and speed, as well as their performance on a sequence of ten serves, and their coaches' comments

group of three students). Each class-period of instruction was led by either one of the teachers or Hyunyi, while both of them observed students' group work. The teachers and Hyunyi reflected on the lesson after class (*Debriefing*), focusing on students' learning processes. After the fourth visit, the teachers analyzed students' written work and identified follow-up instructional tasks, while Hyunyi wrote journal reflections based on an analysis of the audio and video recordings, and students' written work (*Analysis of Student Learning*). The teachers and Hyunyi then repeated this process of choosing, modifying, teaching, and reflecting on two more lessons for the remainder of the visits in the implementation period. The descriptions of MEAs implemented in the teachers' classroom are summarized in Table 4, while the longer versions of the original MEAs are described in other studies (MEA 1: Chamberlin 2005, MEA2: Lesh and Harel 2003, MEA3: Lesh and Doerr 2003).

Data analysis after the implementation

During the 11 weeks of in situ PD, a large corpus of data was collected, as indicated in Table 3. Lesh and Kelly (2000) described a *three-tiered teaching experiment* including possible models developed by students (Tier 1), teachers (Tier 2), and researchers (Tier 3). Student-level models are representationally rich artifacts (e.g., tables, graphs, algebraic or geometric expressions) that reveal students' understanding of the real-world situation; teacher-level models include teaching/assessment tools that represent teachers' understanding of students' thinking, their problem-solving behavior, and their learning needs; and researcher-level models can be the design of PD that represents researcher's understanding of teachers' and students' thinking and behavior (Lesh and Kelly 2000).

This study focuses on the analysis of models developed by the teacher and researcher (Tiers 2 and 3) in order to describe the researchers' analysis of the teacher's learning in the PD situation, along with the teacher's analysis of student learning in the context of MEAs. The analyses of models developed by students (Tier 1) are described more fully in other publications; the first study (Jung 2014) describes students' strategies used for the first and second MEAs, while the second study (Jung 2015) illustrates students' models developed during the third MEA.

To familiarize themselves with the collected data, two authors conducted an open-coding pass (Strauss and Corbin 1998). For each of the two authors, this pass produced a collection of candidate code categories (example codes included: students' mathematical knowledge, students' communication, and teacher's facilitation). After this initial open-coding process was completed independently, the two researchers discussed the categories and merged them. After merging the code categories, the first author sorted out the categories into two groups: (1) the teacher's beliefs and knowledge about students (e.g., students' prior experiences and knowledge; their current thinking and behavior; their mathematical knowledge; their communication; and the difficulties they encountered), and (2) the teacher's instructional decisions and practices (e.g., facilitation, presentation, collaboration, and intentions to implement pedagogical changes). We then repeated the process of individually coding the data and discussing our revised coding. This process helped the authors reach consensus on the main themes within the large corpus of data.

The next step was to document quotations and interpretations from the data related to the research question. To identify roles of the teacher and researcher throughout the in situ PD, we focused on the following contexts: (1) when the researcher initiated the PD, (2) when the teacher shared her concerns, (3) when the researcher responded to the teacher's challenges, and (4) when the teacher reflected on student development. For the first context, where the researcher initiated the PD, each data set was documented based on the six *principles for designing activities for teachers* described in Table 2 in order to describe how the researcher and teacher implemented these principles in the teacher's classroom. For the second context, all the concerns voiced by the teacher throughout the PD were documented. Main themes of these concerns included (a) students' lack of experience in formulating models, validating results, and presenting their modeling process; (b) their unwillingness to work with others; and (c) their lack of prior knowledge of or formal exposure to mathematical concepts (e.g., proportional reasoning, measurement conversions). When the teacher described these challenges, the researcher opened further discussions about how to overcome them. Several quotations from the discussion were documented for the third context (e.g., follow-up questions that the researcher asked to open the discussion). Lastly, the teacher's reflection on student learning and development was documented (e.g., students' capability to present solutions, willingness to collaborate, and capability to learn mathematical concepts through modeling).

Based on these condensed data, three themes (i.e., students' capability to interpret tables, their perceptions of problem solving, and their views about collaboration) were revealed to be common to contexts 2, 3, and 4 (i.e., when the teacher expressed concerns, the researcher responded to them, and the teacher reflected on students' development). A narrative was developed to describe these three themes in each context. Then, the other data sources (e.g., journal reflections, video recordings) were used to verify and enrich the generation of the narrative related to our research question. Finally, the narrative, including the quotations and interpretations, was reviewed by the teacher in a member-checking meeting. This strategy was adopted as a method of triangulating perspectives and balancing the perspectives of participant observers (Jorgensen 1989) with that of an informed outsider. In our analysis, then, we combined three key perspectives: Kate as a participant, Hyunyi as a participant–observer, and the second author as an informed outsider. We shaped our analysis to maximize the value of these three perspectives when combined to build consensus interpretations of the data (c.f., Lincoln and Guba 1985).

Results

Design process in the in situ professional development

When Hyunyi worked with Kate during the in situ PD, Hyunyi proposed that they use the principles in Table 2 to guide their shared thinking. This section describes the practical application of this approach as Hyunyi and Kate collaborated to modify, teach, and reflect on three MEAs in Kate's classroom. We indicate the relevance of each of the six principles in guiding the collaborative design and implementation process.

Multiple contexts principle

As described in Table 2, this principle allows the teacher and researcher to consider the unique qualities of Kate's classroom before implementing a new practice. To meet this principle, Hyunyi learned about the teacher's practice and classroom by observing how Kate planned and taught her lessons for the first and second visits (weeks 1 and 2). In week three, Hyunyi asked Kate's views about students' backgrounds in order for both to consider the diversity of her students (e.g., Please describe your students, including their background and prior knowledge). Additionally, she posed questions related to mathematical contexts of teaching and the classroom environment (e.g., What opportunities do students in your class have to collaborate or to present their mathematical work in class?). These factors guided and enhanced Kate's and Hyunyi's planning process: from their two complementary perspectives, both scrutinized the tasks they designed to ensure they were responsive to the needs and variability of students in each classroom and between the classroom groups of different periods.

Multilevel principle

This principle requires the researcher to engage with teachers in ways that help to develop multiple aspects of the teachers' knowledge, such as mathematical content and pedagogy. During the third visit, Hyunyi introduced ideas from the research literature, about the processes and activities related to modeling, making connections to the topics that the teachers were planning to teach. Hyunyi then asked Kate to work through the three MEAs and select one that she felt would work best in her classes after being adapted and modified. Kate then considered the mathematical learning goals that this MEA would address and reflected on possible difficulties that her students might experience with it. The purpose of these interactions was for the teachers to consider the mathematical pedagogy and content embedded in the proposed MEA. The introduction of the MEAs inevitably positioned Hyunyi as a source of authority, but she mitigated this effect by clarifying that the MEAs she presented were to be taken as "draft" lessons. This invited Kate to engage in adaptation and collaborative modification to move from the received form of the lesson (the initial draft) to the negotiated version that Kate and Hyunyi would actually enact.

Sharing principle

During this third visit, the teachers were also asked to develop a list of items to observe when students worked in groups. This process meets the sharing principle because the teachers knew that the tool that they produced here would be shared with future teachers.

In particular, recognizing the teachers' expertise in this area, Hyunyi asked her permission to use the resulting list in her mathematics education course, where prospective teachers would consider the list in preparing to teach their own lessons. In a similar vein, Hyunyi asked Kate to identify potential follow-up instructional tasks that they would use with their students. This positioned the teachers as authorities on the pedagogical facilitation of classroom tasks; in other words, as an authority on the lesson as it would be enacted. Specifically, Hyunyi said, "I found that your observation list is really helpful, and also the one [follow-up task] that you did for the next day. I would like to use them for my student teachers in the future" [Discussion before MEA 2]. By informing the teachers that the observation list and follow-up task would be used with student teachers, Hyunyi endeavored to ensure that Kate would develop *sharable* products for an authentic, external audience.

Self-evaluation principle

This principle guided the researcher to consider making an environment where the teachers evaluate their teaching goals and practice. To meet this principle, the researcher suggested a de-briefing session after each lesson. In week four, for example, Hyunyi and Kate co-taught a lesson, using the observation list developed during the previous visit. The list was also used to structure part of their de-briefing session after teaching to identify improvisations that had taken place. For example, Hyunyi asked questions including, "What things did you observe other than those on the list?" and "What questions did you ask other than the ones that you listed here?" During this de-briefing session, the teachers described the patterns that they noticed in the students' responses, and they discussed learning goals where they felt their students had had the greatest and the least success. In weeks 5–11 of the study, the teachers and Hyunyi then repeated this process of choosing an MEA, modifying and teaching it, and reflecting on it together.

Reality principle

The whole process described above meets this principle because all the tasks, including lesson planning, teaching, assessment, and de-briefing, were implemented in the teachers' own classrooms. Specifically, the teachers interpreted their own students' work before and after teaching, and they developed assessment tasks based on their analysis of student learning in their classes. This is one of the critical affordances of an in situ PD approach focused on the process of modeling. The teachers' classroom and teaching practice themselves become the context for rich reflections on modeling at both the student and teacher levels.

Obstacles and resources

Throughout this in situ PD, Kate expressed several concerns, including students' capability to interpret data tables, their unfamiliarity with the open-ended nature of modeling problems of MEAs, and the challenges she anticipated with students' working collaboratively in groups. In this section we describe how the researcher opened the discussion for the teachers to share these concerns, along with resources that influenced the teacher's perception of implementing modeling lessons.

Opening the discussion

After the first and second visits, Hyunyi recognized that both the students and teachers were unfamiliar with engaging with modeling lessons that require students to interpret a problem context. Hyunyi asked several questions to elicit concerns that the teachers might have before teaching each lesson, including: “What concerns do you have in mind regarding this draft lesson?” “What do you anticipate students having difficulty with related to this lesson?” and “How can this draft lesson be improved?” At this stage, Hyunyi would again emphasize that each lesson was a “draft” so that the teachers would feel comfortable expressing any anxieties about teaching the newly introduced lesson to their students.

After teaching each lesson, Hyunyi and the teachers had a de-briefing session where Hyunyi opened another discussion to reflect on their lessons. She asked, “What concepts were difficult for students to grasp?” “On which learning goal(s) did the students have the least success?” “Why did you think the students struggled with these goals?” and “If you taught this lesson again, what would you do differently?” Hyunyi asked these questions in part to demonstrate her understanding that students might struggle in their first encounter with a new type of learning task. These questions also allowed the teachers to focus on the reasons for their students’ struggles and consider ways to improve the implementation of the next MEA.

Teacher’s concerns

When Kate solved the *Summer Job* MEA for herself, she expressed a concern that there was too much information in the table for her students to handle (e.g., nine student employees’ number of working hours and amount of money collected in the employer’s store during each month over the summer). She preferred using the readiness problem (e.g., finding a student’s summer earnings when he mows certain yards, wash cars, and get a newspaper route), which included less information in the table (e.g., typical pay for the three different types of summer jobs). When Kate and Hyunyi discussed how to implement this readiness problem, Kate still worried that students would have difficulty in applying the information from tables to solve the problems. For instance, she remarked:

> Just reading tables, taking information from tables and applying that to the problem, I think that is a huge thing for these kids. Because some of them might struggle with reading the table and point that out, putting that to solve the problem.

Kate was also concerned about the open-ended nature of the modeling problems. She thought that her students were accustomed to solving problems that had a “right” or “wrong” answer, and she also anticipated they would have difficulty with pursuing multiple steps to solve a problem. She reported, “They were so used to right or wrong… If there were more than one or two steps that last step they just forgot it. It was brought to their attention by us. And it was almost like a blank dumbfounded look at first.” Kate mentioned that the modeling activities were perhaps the first time she had used a task that had multiple possible correct answers.

Working in groups was another challenge that Kate anticipated, observed, and worked to support her students in overcoming. After teaching MEA 1, she noted that students were more focused on working individually than working together. She surmised that they did not share their answers because they did not see the potential value of collaboration. She remembered that students had asked questions of their group members, such as "What are you doing?" or that they had simply stared at each other and looked around. As Hyunyi listen to Kate's concerns, Hyunyi showed her understanding of these concerns and asked questions focusing on ways to overcome these obstacles. The next section shows ways that Hyunyi acknowledged Kate's knowledge and beliefs, along with how she identified and shared resources to discuss possible approaches to teach MEAs.

Researcher's approach to acknowledging teacher's knowledge and process of modifying the tasks

To address the first concern about students' difficulty in reading tables, Hyunyi understood Kate's anxiety and asked her how she would like to modify the current task for the students. Kate suggested that they change the visual format of the table (e.g., introducing lines between quantities so that students with special needs could easily distinguish each number). She also planned to provide students with an example of how to read the table before receiving the handout. Here, Hyunyi acknowledged Kate's expertise in special education and her knowledge of her own students' capabilities and accepted her suggestions.

For Kate's second concern that the teacher had, which related to the nature of modeling problems, Hyunyi understood the difficulties involved in implementing a problem that has several correct solutions. She agreed that students might struggle with such a task, but she wanted to discuss the importance of modeling. Hyunyi asked questions to elicit Kate's consideration of the task's value, including "Do you think that it's important for them [students] to know that there could be multiple answers?" Kate responded, "I think that kids also need to see in math there isn't always a right or wrong answer. There are situations where there's multiple ways." Seeing the importance of this type of task provided Kate with the motivation to develop instructional strategies to support the new approach. Kate suggested discussing an example problem context with students, in which there were multiple correct answers to the problem. She asked questions of the students (e.g., What if you washed a car for your own parents? How much would you be paid? What if you washed someone else's car?) in order for them to see that their solutions could vary based on what assumptions they made. Students noticed that even though the earned money was different, several solutions could be correct if they performed operations based on their assumptions and provided sound reasoning behind them.

In terms of student collaboration, Kate recalled that some groups had had a hard time working together, and she was concerned about implementing group work. Hyunyi asked her again if there are benefits of student collaboration. Kate mentioned that sometimes students could explain to each other better than a teacher explaining to them because they used "the same lingo" and because students were less afraid of making mistakes when they were working with their peers. Hyunyi then asked how to help the students work better in groups. Kate suggested: "Maybe giving them one worksheet instead [of three or four] would have made them work together, instead of being an individual thing. And just saying, 'I want to see each [of the students'] handwriting on there.'" She thought that it was important for everyone to give input in the MEA work, and she later suggested using a think–pair–share technique to help support the class in doing this. Hyunyi also suggested giving a role to each group member, such as a presenter and a time manager; Kate agreed that having a role would give each student a sense of ownership of their role.

After teaching this lesson, Hyunyi listened to the audio recording of prior de-briefing sessions, watched the video recordings of the MEA 1 implementation, and analyzed the student work from that task. She noticed that Kate often primed her students, providing them with a particular way to solve a problem. Hyunyi understood that it was not easy for a teacher to let students struggle and share multiple solutions in a limited class-time period. However, considering that an important principle behind MEAs is to elicit students' own models without steering their thinking, Hyunyi felt it was important to discuss this matter with Kate. She decided to raise the issue by discussing a teacher guide for MEAs written by other researchers. Hyunyi shared the teacher guide, including the advice to "Avoid questions and comments that steer student's thinking during group work." Kate displayed a high degree of self-awareness, remarking that she had often asked such questions.

Later, the teachers and Hyunyi discussed how they would address the questions that Kate anticipated from her students. Kate assumed that students would ask the following: "What do I use to figure out his height?" and "How do I set up to solve the problem?" Kate's plan took the goal of not directing students' thinking seriously. She remarked, "Pretty much all my answers and responses [to these questions] are guiding the students back to the directions to the problem, 'What do you think?' 'Try to get the other group members to set up and help out.'" Kate's responses show that she valued students' effort to think on their own and learn from other students.

As shown in this section, Kate drew on several resources, including her knowledge of students, her emerging belief in the value of open-ended mathematical problems, and her understanding of students' communication during problem solving. Hyunyi acknowledged Kate's knowledge and teaching experiences and then identified and shared resources to discuss possible approaches to teaching with MEAs. Kate reflected on this experience, the results of her changed instruction, and the factors that helped student development. These are described in the following section.

Kate's reflections on her experiences

Kate's reflections were based on her analysis of her students' learning; she noticed the students' improvement in their capability to solve modeling problems. Specifically, she felt that her students had the greatest improvement in the learning goals of reading tables, problem solving, and working in groups. It seems significant that the areas in which Kate noticed most improvement in her students coincided with the areas in which she had greatest concern about her students' attainment. Thus, Kate came to see the MEAs as an effective experience because they created opportunities to address areas of weakness in her students that she considered important directions for growth.

In terms of reading tables, Kate recognized that the iterative process of engaging with modeling tasks helped students improve their capability to interpret data in tables. She said, "Reading a chart, reading a graph…I definitely saw them get a lot better as we went along, the more we did it." In reflecting on her students' initial perceptions that math problems should have single, correct answers, Kate identified that for her, too, MEAs had required a shift from familiar ways of teaching. She said, "I've always been so black and white with things…There was either right or wrong…But it's definitely something that I'll try to do more of, like definitely letting the kids know that you can think and come up with solutions in different ways." When she changed her perspective on problem solving, she realized her students changed as well. After teaching MEA 3, she reported how her students discussed, reflected on their own, and revised their work.

> I really like how we did it today… I saw whole bunch of kids changing the way that they had originally done it after hearing other kids. And a lot of the final ones that they presented were not a certain kid's in the group—they took all three of theirs and

> mushed them together. So I'm definitely seeing the cycle of everything, of how you start with something, you hear others and you're reflecting, and changing, and like it's a constant, going on, and looking back and knowing that there might not always be a right or wrong answer with stuff.

She also recognized the students' growth in problem solving:

> They definitely are getting a lot better with problem solving. I'm seeing a lot more of the students taking chances that normally don't take chances, that, ah, wanted a right or wrong answer. They are more willing to step outside the box with it, and really try to figure out their own ways.

Lastly, Kate described what she noticed in her students' attitudes related to collaborative work: "I think they finally started to realize that it can be fun, that it is beneficial, and that they actually like it better when they were in groups." She thought that the benefits of working with MEAs were cumulative in this respect, in that students learned from the early MEAs that working and communicating together could be valuable for solving problems.

As described in this section, Kate shared her thoughts about student learning and changes. Her reflection revealed several factors that influenced student development in interpreting tables, solving problems that have multiple solutions, and working in groups. These factors include the iterative process of solving MEAs, the teacher's change in her perception of problem solving, and students' recognition of the value in the collaborative work. When Kate and Hyunyi met for the follow-up visit about a year after the PD, Kate reflected on her continued trajectories of professional growth, which were consistent with some of her thoughts described above, about problem solving and collaborative work. Although she said that she still struggled with creating balanced classroom instruction which incorporates assessments required by the state and the types of problem-solving lessons that she would like to teach, she reported that she had begun letting students solve problems on their owns in groups.

Discussion

In the USA, it is rare that teachers work with university colleagues in their school settings even though this collaboration often improves classroom instruction (Herrenkohl et al. 2010). The researcher–teacher partnership shown in this study demonstrated how such collaboration can be supported by sharing knowledge and resources (Lau and Stille 2014). Through this in situ PD focusing on mathematical modeling tasks, several teacher and researcher roles were highlighted: (1) the researcher's ways of opening the discussions and addressing the teacher's concerns, (2) the researcher's approaches to acknowledging the teacher's expertise, (3) the teacher's strategies for overcoming difficulties, and (4) the teacher's process of reflecting on the factors that helped student development. While the teacher learned about the new mathematical modeling tasks and related research, she helped the researcher recognize classroom realities and implement modeling tasks in these realistic settings. They also shifted roles at different stages of instructional practice (e.g., the researcher led classroom instruction or the teacher analyzed student work), which ensured that both teacher and researcher took "the role of expert" depending on the classroom situation (Lau and Stille 2014).

Unlike traditional studies in education that often theorize concepts about teaching and learning that are isolated from actual lived experiences within the classroom, this study places value on the interconnected relations between research and practice (e.g., Hiebert et al. 2002; Mundry et al. 2000; Wagner 1997). The design of the present study encouraged the participants to implement two related theoretical principles about modeling in the classroom (Doerr and Lesh 2003; Lesh et al. 2000). Following the *principles for designing activities for teachers* (Doerr and Lesh 2003), Hyunyi implemented tasks that required both individuals to consider the diversity of students and classroom environments prior to lesson planning (multiple context). Kate also had the opportunity to solve the modeling problems herself and to discuss how to teach the lessons (multilevel). Such opportunities were used to develop observation lists and assessment tools that could be shared with other teachers (sharing). Finally, Kate also evaluated the teaching goals in her own classroom (self-evaluation and reality). In terms of the *principles for developing MEAs* (Doerr and English 2006; Lesh et al. 1992, 2000), students had the opportunity to engage with MEAs where they collaboratively described, evaluated, and revised their mathematical models (Lesh et al. 2000). This process was difficult for the students because they were not used to interpreting data tables and collaboratively solving real-life problems that have multiple solutions.

Hyunyi worked to promote an environment where the teacher could share her concerns related to such difficulties that students might face. Kate's knowledge was valued when she and Hyunyi had open discussions about how to address the problems in the classroom. As they taught three MEAs, Kate changed her ideas about students' capabilities to interpret a variety of data in tables, to develop problem-solving skills, and to work collaboratively with other students. Her reflective participation in the study involved analyzing her students' work on the MEAs and using this information in the design of the next lesson (Lesh and Kelly 2000). In these exchanges, Kate gained additional perspectives on her students, as well as on their thinking and abilities.

Her reflections show that three factors influenced student development: (1) the iterative process of MEA implementation, (2) changes in her perceptions of problem solving, and (3) developments in her students' views toward collaboration. First, Kate reflected on and expanded her own capacity to help students develop these abilities through three MEAs, and she deepened her sense of the value of doing so. In fact, Kate noticed that these changes in her own practice, in some ways, paralleled her students' development of ideas as they solved real-world problems in the MEAs. Kate reported that prior to this experience, she perceived that there is only one correct answer when solving problems, which her students also believed. When she changed her own view about the nature of modeling and the way of teaching mathematics, she also recognized that her students were beginning to undergo parallel changes. Furthermore, she observed important changes in her students' attitudes: over time, when students found themselves confused, they increasingly sought and provided help effectively, relying increasingly upon each other rather than exclusively on the teacher. Once Kate changed her perspectives, she was able to ask questions that led students to discuss multiple ways to solve the problem and justify their solutions—all of which are aligned with the *design principles of modeling activities* (Doerr and English 2006; Lesh et al. 1992, 2000).

Conclusion

Several researchers stated two main reasons why education research is often criticized: (1) education research cannot generate studies to improve classrooms without teachers' active participation, and (2) traditional educational studies treat teachers as the "object of research" rather than "an integral part of the research process," which reflects power differentials (Edwards and Jones 2003, p. 137; Wagner 1997). This study emphasizes a teacher's active involvement in the research-teaching process. For example, the teacher selected modeling tasks for her classroom, shared concerns, and offered instructional strategies that would help her students effectively solve modeling problems. She also collaboratively modified and taught the lessons, analyzed the students' work, and collaboratively reflected on the factors that helped student development. More institutional support for such collaboration is needed, especially when teachers are not usually involved in this intense process due to institutional challenges and time constraints (Herrenkohl et al. 2010; Lau and Stille 2014). In spite of these limitations, several studies show the promise of researcher–teacher collaboration for enhancing student learning and expanding the research literature (e.g., Edwards and Jones 2003; Herrenkohl et al. 2010; Lau and Stille 2014).

Challenges also occur in teachers' classrooms when implementing new types of tasks. To address the difficulties that could possibly occur in other classrooms, teacher educators may also consider the concerns the teacher expressed in this study (e.g., students' potential difficulties in reading the real-life data in tables, solving problems that have multiple correct answers, and collaborating with each other). The researcher recognized and validated the teacher's concerns, acknowledged her experiences, and oriented the discussion to consider the value of modeling tasks. The researcher also encouraged the teacher to share her instructional strategies and provided resources that could help her see different perspectives about teacher roles. By discussing instructional practice and analysis of student learning, the researcher acquired more insight into classroom approaches and collaborative research processes (Edwards and Jones 2003). The narrative nature of this study enables the readers to see specific ways in which the community of teachers and researchers work together as partners when implementing new tasks in the classroom (Hiebert et al. 2002). The study supports the value and viability of this model of in situ professional development, indicating that significant changes in teachers' thinking about their students' model development can occur in relatively short periods of time. Researcher–teacher partnerships can enrich classroom learning and provide opportunities for researchers and teachers to develop their analysis of students' mathematical thinking.

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