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# Precision Measurement of the Electronic $g$ Factors of the Alkali Metals*

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We have measured the ratios of the electronic $g$ factors of $^{23}$Na, $^{85,87}$Rb, and $^{133}$Cs to that of $^{39}$K, using the atomic-beam magnetic-resonance technique. The results are $g_J(^{23}\mathrm{Na})/g_J(^{39}\mathrm{K})=1.0000007(2)$, $g_J(^{85,87}\mathrm{Rb})/g_J(^{39}\mathrm{K})=1.0000182(2)$, and $g_J(^{133}\mathrm{Cs})/g_J(^{39}\mathrm{K})=1.0001231(3)$. These results, combined with the results of other researchers, yield the following absolute $g$ factors for the alkali metals: $g_J(\mathrm{Na})=-2.002297(2)$, $g_J(\mathrm{K})=-2.002295(2)$, $g_J(\mathrm{Rb})=-2.002332(2)$, and $g_J(\mathrm{Cs})=-2.002542(2)$. These $g$ factors, with the exception of the value for Cs, are in good agreement with theoretical values.

## I. INTRODUCTION

IN the investigation of the hyperfine structure (hfs) of free atoms by the atomic-beam magnetic-resonance technique, it is necessary to measure both the frequency of the radio-frequency (rf) magnetic field causing a transition between two hyperfine energy levels, and the strength of the static magnetic field in which the transition takes place. Often both must be determined with the same precision. Calibration of the field is most easily accomplished by observing a transition in a beam of alkali-metal atoms. The frequency of the alkali-metal transition then allows one to calculate the static magnetic field, provided the constants describing the hfs are accurately known. The measurements described here were made to provide a precise, consistent set of electronic $g$ factors $g_J$ for the alkali metals sodium, potassium, rubidium, and cesium. The other constants necessary to describe the hfs of these metals are well known from other work.

## II. THEORY OF THE EXPERIMENT

The electronic ground state of all the alkali metals is $^2S_{1/2}$, arising from a configuration of one $s$ electron outside closed shells. The Hamiltonian describing the hfs of this state is

$$\mathcal{H}=ha\mathbf{I}\cdot\mathbf{J}-g_I\mu_0\mathbf{I}\cdot\mathbf{H}-g_J\mu_0\mathbf{J}\cdot\mathbf{H}, \quad (1)$$

where $a$ is the hfs magnetic-dipole interaction constant, $I\hbar$ is the nuclear angular momentum, $J\hbar$ is the electronic angular momentum, $g_I=\mu_I/I$ and $g_J=\mu_J/J$ are the corresponding $g$ factors, $H$ is the magnetic field, $h$ is Planck's constant, and $\mu_0$ is the Bohr magneton. The energy levels of Eq. (1) are given by the Breit-Rabi formula,

$$W(F,m_F)=-h\Delta\nu/2(2I+1)-g_I\mu_0m_FH+(F-I)h\Delta\nu[1+4m_Fx/(2I+1)+x^2]^{1/2}, \quad (2)$$

where $\Delta\nu=a(I+\frac{1}{2})$, $x=(g_I-g_J)(\mu_0/h)H/\Delta\nu$, and $\mathbf{F}=\mathbf{I}+\mathbf{J}$.

The theory of the operation of an atomic-beam apparatus has been described in detail elsewhere.[1] (We describe below those features unique to this experiment.) For present purposes it is sufficient to say that any transition between two hfs levels that satisfies the usual selection rules, and in addition satisfies the condition that $\Delta m_J=\pm1$, can be observed by using an atomic-beam apparatus.

At high magnetic fields ($x\gg1$) the frequency corresponding to the transition $(F=I+\frac{1}{2},\ m_F=-I+\frac{1}{2})\leftrightarrow(F=I+\frac{1}{2},\ m_F=-I-\frac{1}{2})$ is given approximately by

$$\nu=-aI-g_J(\mu_0/h)H. \quad (3)$$

This means that $[\partial\nu/\partial g_J]$ is of the order of $10^3$ at magnetic fields of a few kilogauss, and observations of this transition can provide a precise determination of $g_J$.

## III. THE EXPERIMENT

### A. Experimental Procedure

Atomic beams of sodium, potassium, rubidium, and cesium were obtained by heating a mixture of fresh calcium filings and either NaCl, KCl, RbCl, or CsCl in an oven. The oven was resistance heated and had a 0.005-in. slit. These beams were detected by allowing them to impinge on a hot iridium wire. The ions evaporated from the wire were collected, and the ion current was measured by use of a picoammeter. Ion currents of $10^{-10}$ A were typical.

The procedure for an experimental run was as follows: First, the standard transition $[(F=I+\frac{1}{2},\ m_F=-I+\frac{1}{2})\leftrightarrow(F=I+\frac{1}{2},\ m_F=-I-\frac{1}{2})]$ was observed in $^{39}$K, then in the other alkali metal under investigation, then again in $^{39}$K, etc., until approximately 25 observations had been made for each isotope. A single observation consisted of the average of six recorded frequencies. These six frequencies consisted of three pairs of frequencies symmetrically placed about the peak of a resonance, ranging from somewhere near

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[1] N. F. Ramsey, *Molecular Beams* (Oxford University Press, N. Y., 1956).

TABLE I. Result of least-squares fit of data to Breit-Rabi equation.

| | $H(G)$ at which measurements were made | Value assumed for $g_J(^{39}K)$ | Total number of observations | $\chi^2$ | Least-squares $g_J$ results |
|---|---|---|---|---|---|
| $^{23}Na$ | 1500, 2411, 3300, 3550, 3750, 3950, 4200 | −2.00229800 | 256 | 85.8 | −2.00229898(17) |
| $^{85}Rb$ | 1800, 2700, 3600, 4500 | −2.00230900 | 122 | 8.2 | −2.00234556(20) |
| $^{87}Rb$ | 1800, 2700, 3600, 4500 | −2.00230900 | 122 | 3.7 | −2.00234550(20) |
| $^{133}Cs$ | 900, 1300, 1900, 2400, 3000, 3700, 4400 | −2.00230900 | 314 | 21.2 | −2.00255541(33) |

the peak to somewhere near half-maximum. All this was done as rapidly as possible to eliminate effects of magnetic field drift. Ordinarily we could switch from one alkali metal to the other in about 2 min.

The measurements were made at a variety of magnetic-field strengths varying from 1000 to 4500 G, with the strength used at any one time depending on the alkali. A nuclear-magnetic-resonance probe and feedback device was used to regulate the magnetic field. Field stability was typically within a few parts per million.

### B. Radio-Frequency Equipment

The basic oscillator for this experiment was the Schomandl Model FD3. For frequencies within the 300-MHz to 1-GHz range of this oscillator, the signal was simply amplified by an Electro-International Model AP-502R triode amplifier before being sent to the beam apparatus. This range could be extended to 3 GHz and beyond if necessary by using crystal multiplication and traveling-wave-tube amplifiers. From 3.3 GHz to 12.4 GHz we used klystrons (Sperry 2K42, 2K43, and 2K44; and Varian X-13 and X-13B) phase-locked to some harmonic of the Schomandl oscillator frequency by either a Dymec Model 2650A Oscillator Synchronizer or a Schomandl Model FDS-3 Syncriminator. All frequencies in the experiment are referred to atomic time A1, which defines the second by taking $\Delta\nu(^{133}Cs) = 9192.631770$ MHz exactly.

In the earlier phases of this experiment, the rf loop or hairpin shown on the left in Fig. 1 was used. It consists of a standard 50-Ω coaxial line that has been flattened at the end so that it tapers gradually into a short. However, subsequent work revealed that this hairpin introduced systematic errors into the results. Figure 2 shows two graphs, one of the quantity $R=[g_J(^{133}Cs)/g_J(^{39}K)]-1$ measured at various magnetic fields with this shorted hairpin, the other showing the same quantity $R$ obtained as a function of magnetic field with the hairpin illustrated on the right in Fig. 1. $R$ is expected to be independent of magnetic field. The scatter of the results for the shorted hairpin probably results from peculiar standing-wave patterns set up in the tapered section at high frequencies. The power for the potassium frequency and the power for the cesium frequency may have maximized at different locations; because the magnetic field was not perfectly homogeneous, the measurements on the two isotopes were thus not made at the same magnetic field, resulting in an error when a comparison was made. A hairpin designed

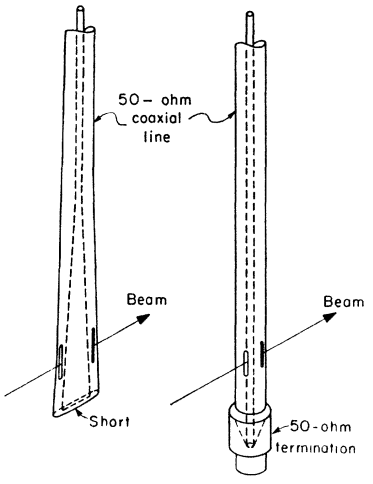


FIG. 1. Sketch of the shorted and terminated types of hairpins used to generate the radio-frequency fields.

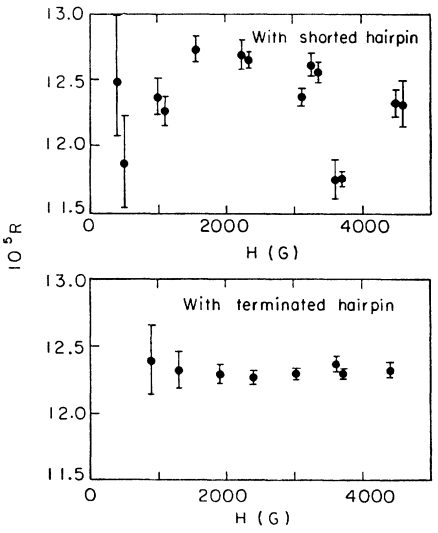


FIG. 2. Comparison of the performance of the shorted and terminated types of hairpins at high magnetic fields and frequencies. The quantity $R=[g_J(^{133}Cs)/g_J(^{39}K)]-1$ is plotted against $H$.

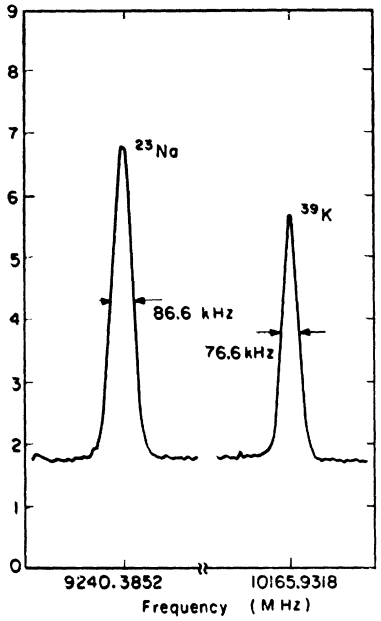


FIG. 3. Chart-recorder tracings of the standard transition in $^{23}$Na and $^{39}$K at 3750 G.

to eliminate this effect, illustrated on the right in Fig. 1, consisted of a standard 50-Ω coaxial line terminated in a matched load. As can be seen from Fig. 2, this modified hairpin yielded much more consistent results.

### C. Data

Chart-recorder tracings of the resonances observed in this experiment are shown in Figs. 3–6. The lines are very free from structure and are quite symmetric. To check for a possible Millman effect in our hairpin,[2] we made at least one run for each alkali with the static magnetic field reversed. No effect was observed in the results.

TABLE II. Constants assumed for the alkalis in the least-squares-fit calculations.

| | $\Delta\nu$ | $g_I \times 10^{4}$[a] |
|---|---|---|
| $^{23}$Na | 1771.626047(100)[b] | 8.04639[c] |
| $^{39}$K | 461.719723(30)[d] | 1.41922[c] |
| $^{85}$Rb | 3035.732439(5)[e] | 2.93700[c] |
| $^{87}$Rb | 6834.68614(3)[e] | 9.95323[c] |
| $^{133}$Cs | 9192.631770[f] | 3.98994[c] |

[a] This assumes $\mu(H^1)_{\text{uncorr}} = 2.792670$ nm.
[b] Y. W. Chan, V. W. Cohen, M. Lipsicas, and H. B. Silsbee, Phys. Rev. **150**, 933 (1966).
[c] I. Lindgren, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Co., Amsterdam, 1965), Vol. 2, p. 1621.
[d] S. Penselin (Institut für Agewandte Physik der Universität Bonn, Germany, private communication).
[e] S. Penselin, V. Moran, V. W. Cohen, and G. Winkler, Phys. Rev. **127**, 524 (1962).
[f] The $^{133}$Cs $\Delta\nu$ is the present frequency standard.

[2] S. Millman, Phys. Rev. **55**, 628 (1939).

TABLE III. Final $g_J$ results.

$g_J(^{23}\text{Na})/g_J(^{39}\text{K}) = 1.0000007(2)$
$g_J(^{85,87}\text{Rb})/g_J(^{39}\text{K}) = 1.0000182(2)$
$g_J(^{133}\text{Cs})/g_J(^{39}\text{K}) = 1.0001231(3)$

## IV. RESULTS AND DISCUSSION

### A. Results

The data were analyzed as follows. The average of the two potassium frequencies bracketing a particular frequency for the other alkali metal involved was used to calibrate the magnetic field for that alkali frequency. A least-squares fitting routine was used to fit the Breit-Rabi formula to the data. A value was assumed for $g_J(^{39}\text{K})$ and the $g_J$ of the other alkali was varied until the best fit was obtained. Table I gives the results of the least-squares fit, and Table II lists the hfs constants that were assumed for the alkalis in the calculation. The low values of $\chi^2$ indicate that our choices of uncertainties for the data were conservative. However, these choices reflect considerations of possible systematic errors.

By dividing the results of the least-squares fit by the value of $g_J(^{39}\text{K})$ assumed, we obtain the quantity actually determined by this experiment, namely the ratio of $g_J(^{39}\text{Cs})$, $g_J(^{87}\text{Rb})$, $g_J(^{85}\text{Rb})$, or $g_J(^{23}\text{Na})$ to $g_J(^{39}\text{K})$. Final values for these ratios, with errors increased to two standard deviations, are given in Table III. The average value of the ratios $g_J(^{87}\text{Rb})/g_J(^{39}\text{K})$ and $g_J(^{85}\text{Rb})/g_J(^{39}\text{K})$ is listed there as $g_J(^{85,87}\text{Rb})/g_J(^{39}\text{K})$, since the two values agree extremely well, and White *et al.* report $g_J(^{85}\text{Rb})/g_J(^{87}\text{Rb})$

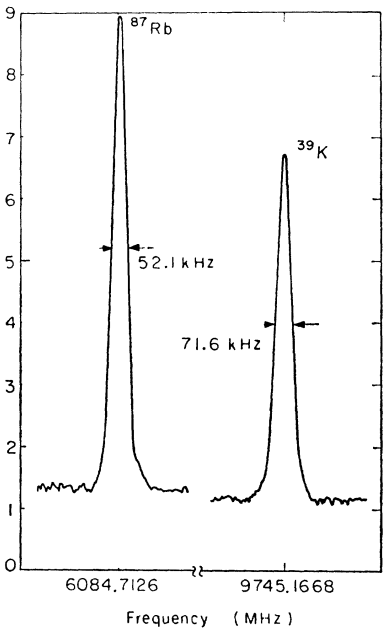


FIG. 4. Chart-recorder tracings of the standard transition in $^{87}$Rb and $^{39}$K at 3600 G.

TABLE IV. Comparison of theoretical and experimental $g_J$'s.

| | $(\Delta g)_M$ | $(\Delta g)_L$ | $(\Delta g)_{CM}$ | Theory | Experiment |
|---|---|---|---|---|---|
| Na | 0.000022 | 0.000004 | −0.000002 | −2.002295 | −2.002297(2) |
| K | 0.000020 | 0.000004 | −0.000005 | −2.002300 | −2.002295(2) |
| Rb | 0.000020 | 0.000004 | −0.000050 | −2.002345 | −2.002332(2) |
| Cs | 0.000018 | 0.000004 | −0.000140 | −2.002437 | −2.002542(2) |

$=1.00000000(1)$.[3] The results of Table II agree with our previously published results,[4] and are also in excellent agreement with values obtained independently by Böklin *et al.*[5]: $g_J(^{23}\mathrm{Na})/g_J(^{39}\mathrm{K})=1.0000012(5)$, $g_J(^{85,87}\mathrm{Rb})/g_J(^{39}\mathrm{K})=1.0000184(4)$, and $g_J(^{133}\mathrm{Cs})/g_J(^{39}\mathrm{K})=1.0001228(3)$.

### B. Discussion

It can be shown that because of the simplicity of the electronic ground-state configuration all second-order corrections to the $g_J$ obtained by the Breit-Rabi formula vanish.[6] Hence, our least-squares results give us the true $g_J$ of the alkali-metal atoms involved.

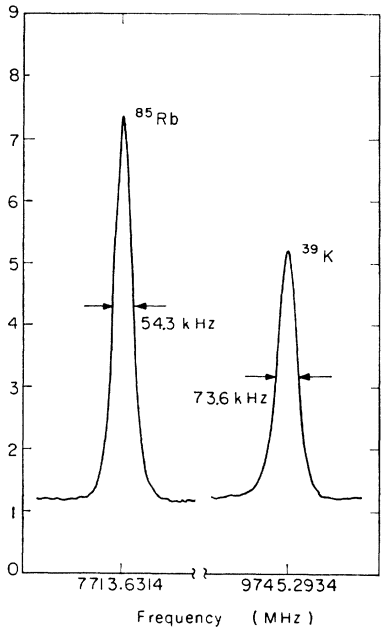


FIG. 5. Chart-recorder tracings of the standard transition in $^{85}$Rb and $^{39}$K at 3600 G.

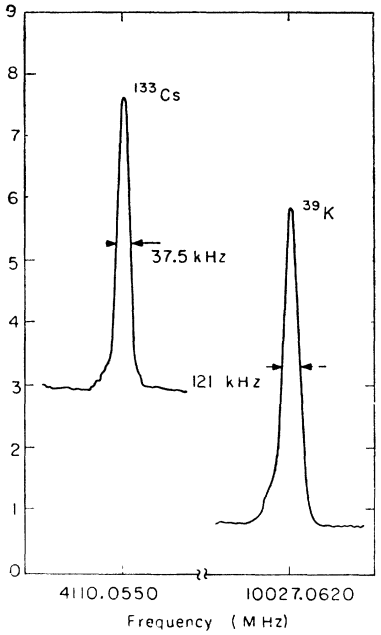


FIG. 6. Chart-recorder tracings of the standard transition in $^{133}$Cs and $^{39}$K at 3700 G.

From our results we can obtain absolute $g_J$ factors for the alkalis by using Balling and Pipkin's measurement[7] of $g_J(^{85}\mathrm{Rb})/g(e)$ to connect our measurements to Crane's determination of the absolute $g$ factor of the free electron.[8] Using $g_J(^{85}\mathrm{Rb})/g(e)=1.0000063(10)$ and $g(e)=-2.002319244(54)$, we obtain the experimental values given in Table IV. Also listed in Table IV are the theoretical corrections to the free-electron $g$ factor caused by atomic effects, as calculated by Phillips[9] and Perl.[10] The three corrections listed there are: $(\Delta g)_L$, the correction for diamagnetism discussed by Lamb[11]; $(\Delta g)_M$, the relativistic correction discussed by Margenau[12]; and $(\Delta g)_{CM}$, the configuration-mixing correction calculated by Phillips.[9] The sums of the free-electron $g$ factor $(-2.002319)$, and $(\Delta g)_L$, $(\Delta g)_M$, and $(\Delta g)_{CM}$ are listed as the theoretical $g_J$ for the four alkalis. The comparison with experiment is good except for cesium. Only $(\Delta g)_{CM}$ is of the right magnitude and sign to account for the discrepancy; our results indicate that this correction should be recalculated for cesium.

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