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THE PROBLEM OF MANAGING A STRATEGIC RESERVE

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Abstract—We develop a method of managing strategic reserves, in this case, the U.S. Strategic Cobalt Reserve. A rationale for the stockpiling of cobalt is presented, followed by a method for bringing the stockpiled amount from any level to a desired goal, in this case an amount determined by the Federal Emergency Management Agency to last through three years of conventional warfare. The method involves solving a stochastic programming problem in order to balance the expected values of the social benefit and the social cost of building the stockpile—Social benefit is accrued by decreasing the impact and the probability of a war or a major supply disruption occuring before the stockpile goal is realized; social cost is determined from the additional amount U.S. cobalt consumers must pay due to the increase in world demand brought about by stockpiling. The management of the filled stockpile is then discussed, introducing the idea of using the stockpile to assure stability in the world price of cobalt and of defraying maintenance costs by market speculation. Least-squares fitting is used to determine whether prices are high or low and how much to sell or buy, respectively, to bring prices back into line. Then the conditions under which the stockpile should be drawn down are considered, with the proper rate and total amount of released stockpile material determined for two cases, that of a major supply disruption and that of actual warfare. Finally, generalization of the method to cover other strategic stockpiles is discussed.

INTRODUCTION

In 1946 the Federal Government decided to establish stockpiles of certain minerals which were deemed important for national defense and which were, for the most part, imported from other countries. The metal *cobalt* is one material which has been stockpiled since the program's inception. It is used in many important industries, and is no longer mined or produced domestically. We will investigate some of the considerations in creating and maintaining a stockpile of the strategic metal cobalt as an example of the general problems of stockpile management.

We begin by showing the current necessity of a cobalt stockpile, due to the many uses of cobalt and the current vulnerability of its supply. We continue by developing a model for filling the stockpile and managing the stockpile once it has reached its capacity. We then consider the criteria and methods for directing the draw-down of the stockpile during crisis situations. Finally, we discuss extentions of these methods to other materials.

THE PROBLEM OF VULNERABILITY

Cobalt is used in many high-technology applications: 'superalloys' for aviation technology, cementing compounds for high tensile strength tools, magnets in electrical equipment, dryers and pigments in paints, and so forth. Many of its applications have strategic value in a possible conflict; in 1980, defense used 2.9 million pounds or 17 percent of that year's U.S. consumption, and during the Vietnam war, cobalt consumption increased 39 percent. Unfortunately for the above applications and for national defense, there is no domestic production of cobalt[2, 5, 7].

These facts indicate that the United States is vulnerable to disruptions in the supply of cobalt. There are two primary concerns: disruptions caused by problems in supplying countries, and those caused by war.

The first, shortages and price increases due to a upheaval in an unstable source of supply, has already taken place in the 1970s. In September 1976, the Federal Emergency Management Agency (FEMA), which oversees the stockpile program, increased the goal for the cobalt stockpile and ceased selling what had been a surplus of the metal. This caused a tight market and some price increases. Then, in 1978, a political uprising in Zaire brought production in that country to a halt and caused a panic. Prices jumped from \$2.00 per pound to \$25.00 with prices of \$50.00 per pound on the spot market. The problem in Zaire was resolved in less than a year, but prices of \$25.00 per pound continued into 1981[8].

The second type of disruption is the type the Strategic and Critical Materials Stockpiling Act was intended to mitigate—disruption due to severed supply lines and increased demand during wartime. Currently, according to the provisions of the Act, this is the only legitimate use of the U.S. stockpile; we, however, believe that there is also merit to using the stockpile to alleviate the first type of disruption.

Other concerns are the possibilities of a cobalt cartel, the abilities of supplying countries to meet future increases on demand, and the almost inevitable prices increases as accessible sources are used.

APPROACHES TO DECREASE VULNERABILITY

The creation of a strategic cobalt stockpile is not the only means of solution that has been suggested for the above problems. In fact, for the long-range problems of availability and price, a stockpile is at best useless and at worst a means of treating symptoms if it is drawn down during the beginning stages of a significant problem. Such long-range problems require long-range solutions.

During and after the Zaire crisis, there was considerable research into alternate solutions. The U.S. has 100 million pounds of proven cobalt reserves which could be brought on line to produce six to eight million pounds per year if the mine operators were given a three to four year lead time and a guaranted price of \$25 per pound for the next ten years. Recycling and less wasteful manufacturing techniques resulted in some savings after 1979 and could result in further savings if the producer prices for cobalt increase. Research to discover and improve materials which can substitute for cobalt and are more readily available, and research into ocean floor mining may also provide help for the long-range picture[2, 5].

None of these solutions are available on short notice, however. Domestic reserves, recycling, and substitute can all be considered stockpiles which require a certain initial expenditure and lag time before they can be 'drawn down.' Also there is a cost associated with drawing them down so that they are only worth using if the price of cobalt rises over a certain line for an extended period of time.

Thus, some short-term solution is required for the disruption caused by war or an unstable supply; a stockpile is well-suited to those cases. Currently the stockpile size goals are set by FEMA. They use an interdepartmental procedure to calculate the requirements of a wartime industry for strategic materials during a three-year war fought with conventional (non-nuclear) weaponry. They do not make their criterion publicly available for security reasons; thus, as mathematicians, all we can do is accept their findings and work with them. The present stockpile goal is 85.4 million pounds, 45.9 million of which had been stockpiled by 1983[8].

REACHING THE STOCKPILE GOAL

Perhaps the biggest concern of stockpile management is that of insuring that the stockpile is of adequate size; in this case, enough cobalt to last through a three-year conventional war as determined by FEMA. At present, the stockpile of cobalt is too low and needs to be built up. In the future, stockpile requirements may be changed at any time. Stockpile management requires a means of determining the rate at which cobalt should be bought.

The simplest method is that of purchase at a constant rate. If A is the amount of cobalt needed to be purchased and t is the time till target date (the date determined when the stockpile should have reached its goal), then R, the rate at which cobalt should be bought, is simply

$$R = \frac{A}{t}. \quad (1)$$

Thus, if the current month is February 1985 and the current stockpile is at 51.4 million pounds of cobalt, while the target stockpile is 85.4 million pounds and the target date is December 1991, then the amount of cobalt needed A = 34.0 million pounds of cobalt and time t = 83 months. In this case, the recommended rate of purchase is R = 34/83 million = 410 thousand pounds of cobalt per month (4.92 million pounds per year).

A constant purchasing rate is the simplest, and, for a first approximation, the best, since a constant purchasing rate keeps the market as steady as possible and reduces the chances of undesirable fluctuations in price (due to fluctuating demand). Some reasons and models for purchase and sales of cobalt at non-constant rates are given below.

It should be noted that this model works more generally than just for initial stockpile buildup. Suppose the target stockpile of cobalt has been reached. If, for any reason, the required amounts of stored metal is increased, then this new target, along with a new target date, gives a new purchasing rate. If, on the other hand, the required reserve amount *decreases*, then A is negative, R is negative, and formula (1) gives the rate at which cobalt should be *sold*. During stockpile buildup, any time a purchase above or below the constant rate is made, a new rate should be calculated.

BENEFITS OF ACCELERATED PURCHASING

If there is a goal stockpile level, then what is the optimum rate at which to reach that level? We wish to answer that question by looking at a *stochastic programming problem*. We want to minimize, subject to certain constraints, the expected value of the cost of a solution in which certain costs have an associated probability distribution for their actual occurrence. Specifically, there will be costs associated with having a shortfall in the stockpile if a war or disruption should occur which should be weighted by the probability of such a disruption actually occurring. Only if we *knew* (p = 1) that a war would occur would we build up at a very high rate and risk disrupting the market.

We can simplify our problem by breaking the possible future events into three cases (more could be used): war disruption, supply or price disruption, and no disruption. Assigning a probability to each: p_w, p_s, and $p_n = 1 - (p_w + p_s)$ respectively, our stochastic programming problem becomes a nonlinear programming problem. We wish to minimize $p_w c_w + p_s c_s + p_n c_n$, where c represents the costs associated with war, supply disruption, and no disruption.

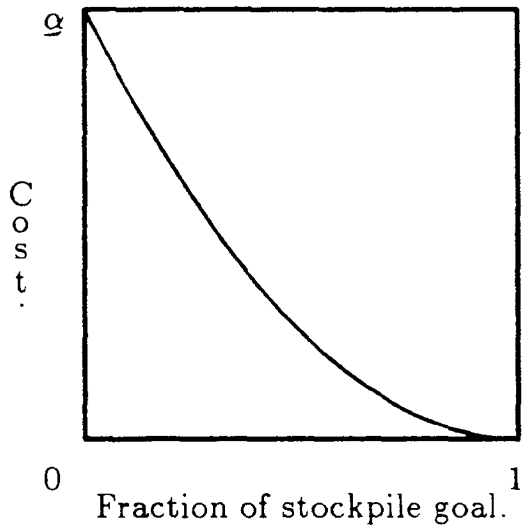

Fig. 1. Cost of a disruption vs amount stockpiled.

Let us consider the social cost of a disruption as a function of the fraction of stockpile complete. Thus if s is the stockpile level and g is the goal, cost $= \xi(s/g)$. We would expect the following to hold:

$$\left.\begin{aligned} \text{Domain of } \xi &= [0, 1] \\ \xi(0) &= \alpha \gg 0 \\ \xi(1) &= 0 \end{aligned}\right\} \quad \text{and} \quad \left\{\begin{aligned} &\xi'(x) \leq 0 \text{ for } x \in [0, 1] \\ &\xi'(0) < -1 \\ &\xi'(1) = 0 \end{aligned}\right.$$

where α is the cost with no stockpile. The expectation that the cost with a full stockpile is zero is unrealistic, but we can move the axis to give us this zero for easier computations. That is, we expect the curve to look like Fig. 1; the closer the stockpile is to the goal, the less expensive a disruption will be.

Integrating the cost over one year, we obtain Eq. (2) for the yearly cost of a buildup in which the stockpile contains *level(t)* pounds of cobalt at time t.

$$\int_0^1 \xi \frac{\text{level}(x)}{g} \, \mathrm{d}t. \tag{2}$$

For a specific example, let $\xi(x) = \alpha(1 - x)^2$ and consider buying at a constant rate m. Then the level at time t, level$(t) = mt + s$, and the cost for one year,

$$\begin{aligned} \int_0^1 \xi\left(\frac{\text{level}(t)}{g}\right) \mathrm{d}t &= \int_0^1 \alpha\left(1 - \frac{mt + s_0}{g}\right)^2 \mathrm{d}t \\ &= \frac{\alpha}{g^2} \int_0^1 (g - s_0 - mt)^2 \, \mathrm{d}t \\ &= \frac{\alpha}{g^2}\left((g - s_0)^2 t - m(g - s_0)t^2 + \frac{m^2 t^3}{3}\right)\Bigg|_0^1 \\ &= \frac{\alpha}{g^2}((g - s_0)^2 - m(g - s_0) + m^2/3). \end{aligned}$$

In terms of m, this is a parabola with minimum at $m = 3(g - s_0)/2$. From this, it is clear that increasing the rate of cobalt purchase beyond the constant purchase rate will decrease the potential cost for a disruption. In fact, considering only this equation, the best plan is to buy all the cobalt required immediately. We can also use nonlinear functions for a buying rate, and approximate any arbitrary buying function with a piecewise-linear function. The factor of $(g - s_0)^2$, which is very large when the stockpiled amount is small, indicates that it is important for us to buy quickly when the stockpile is low and it less urgent when it is nearly filled.

Both disruption functions, c_w for war and c_s for supply disruptions, would have the above form, though the maximum cost for supply disruptions will be less than that for war. For example, the cost of a disruption has been estimated at $\$1.8 \times 10^9$ and its probability between .3 and .7 for the 1980's[7]. We will use a probability of .05 for a supply disruption occurring in a given year. Furthermore, if we estimate the costs and probabilities of war at $\$2 \times 10^{10}$ and .005 in a year, our total weighted cost for a 5 million pound a year buying rate is about $32.6 million. This cost must be balanced against the costs of buying too much and *not* having a crisis.

COST OF ACCELERATED PURCHASING

The upper limit on the amount of cobalt that can be purchased at any time is computed from the social cost of making that purchase. We define the social cost as the amount extra that U.S. companies have to pay to get their cobalt because the government stockpile purchasing has created an artificial demand for cobalt. To determine this social cost, we need to know the dependence of price on demand. In economics, this is usually termed the *price elasticity of demand,* and is computed thus:

$$-\eta = \frac{P}{D \cdot \dfrac{dP}{dD}}, \tag{3}$$

where η is the elasticity, P the price, and D the demand.

Solving this differential equation for P gives us

$$-\eta \frac{dP}{dD} = \frac{P}{D}$$

$$\frac{dP}{P} = -\frac{1}{\eta}\frac{dD}{D}.$$

Integrating provides:

$$\begin{aligned} \ln(P) &= -\frac{1}{\eta}\ln(D) + C \\ P &= C \cdot D^{-1/\eta}. \end{aligned} \tag{4}$$

In our application, we want to know only what happens to the price when demand is increased. Thus, we need an expression for P if D is replaced with $k \cdot D$:

$$\begin{aligned} P' &= C \cdot (kD)^{-1/\eta} \\ P' &= k^{-1/\eta} \cdot CD^{1/\eta} \\ P' &= k^{-1/\eta} \cdot P. \end{aligned} \tag{5}$$

With this equation, the increase in price due to stockpile demand can be calculated—if demand is increased by 20 percent, then the new price is $1.2^{(-1/\eta)}$ times the old price. The social cost of buying the cobalt for the stockpile is consequently the current U.S. demand times the difference in price, or $P' - P$. One note needs to be made here: the social cost of buying is predicated on the assumption that the original price is the price without any stockpile buying. Thus, if we are in the second year of our stockpiling spree, the price already reflects earlier buying, and to calculate the social cost requires determining what the market price would have been if cobalt had not been bought for the stockpile.

For example, let us assume that we wish to purchase 5 million pounds of cobalt for the stockpile this year, at a price of $5.10 per pound. Since last year the U.S. government purchased 6.5 million pounds[8], we first have to correct the world price. This is done by setting $k = 0.892$ in the above equation, since that is the percentage of world output that was not stockpiled, based on an estimate at the world production of cobalt of 60 million pounds. The long-term price elasticity of demand for cobalt has been found to be -0.32[2], so:

$$P = (0.892)^{-1/-0.32} \cdot \$5.10 = \$3.57. \tag{6}$$

Thus the beginning world market price we will use is $3.57 per pound. Now we wish to buy 5 million pounds. This represents an increase of 9.35 percent over the previous world demand of 53.5 million pounds, so the price will change as follows:

$$P' = (1.0935)^{-1/-0.32} \cdot \$3.57 = \$4.72. \tag{7}$$

The social cost of this buying for the stockpile is just the price difference times U.S. demand, which for 1985 is predicted to be 25 million pounds. Thus the cost is

$$25 \text{ million} \cdot (\$4.72 - \$3.57) = \$28.75 \text{ million}. \tag{8}$$

Contrast this figure with the $32.6 million social benefit found above. Clearly we are buying at a rate for which the social benefits outweigh the costs; in fact, we could buy at a faster rate without real damage to the U.S. economy, and probably wish to do so for reasons of national security.

What would happen if the social benefit was less than the social cost of a buying rate? This would happen only if the rate at which we are buying is too high, and would be a clear signal that we should immediately move to a lower rate of purchase. Notice, however, that our model always begins with a linear purchase rate, and that it is possible that the costs even of this base rate are higher than the benefits. If so, then the model has indicated that the original goal is unreasonable—it hurts the economy more than it helps, even if a war is immiment!

Now this seems to be a problem. What if war is immiment? Should we buy at a rate which the model shows to be economically damaging so we can survive the war, or is the model wrong and the rate actually reasonable, given the risk of war? The answer is that if war is really that imminent, and we still found a cost/benefit imbalance, that we probably did not use the correct probability of war in the benefit analysis. We ought to use a probability very close to one, and if we redo the analysis with such a high probability, we may find the rate is not unreasonable after all. If not, if we have put all the best data into the model and we still get a cost/benefit imbalance, then we really do have an unreasonable and unreachable goal.

In general, though, the algorithm for determining how much cobalt to buy is one which maximizes expected benefit by stockpiling before a crisis while minimizing the social cost to the U.S. economy. The point of intersection of the benefits and costs is the rate at which we wish to buy cobalt, for we do the most good and least harm to the U.S. economy.

PRICE FLUCTUATIONS AND STOCKPILE MANAGEMENT

Until now, our mathematical model has been largely concerned with the building up of the stockpile. As such we have not explicitly dealt with the problem of price fluctuations in the market. Cobalt prices have long-range trends and short-term fluctuations. Presumably, it is desirable to purchase cobalt when the price has swung low, or before long-range pricing trends increase the cost.

The desirability of purchasing extra cobalt when the price is low is, however, already reflected in our model. When the price is low the net social benefit of buying extra cobalt due to disruption probabilities does not change. At the same time, the net social *cost* of purchasing cobalt is reduced, and so extra cobalt would be bought when the price has fluctuated low simply because it would be less harmful to the economy to do so when the price is lower.

Once the stockpile has reached its goal, we are faced with a different question. We can let the stockpile sit until a crisis occurs, or we can buy and sell from the stockpile. It would be nice to have the stockpile pay for its own maintenance costs by buying when the price is low and selling when the price is high. There are also economic benefits in keeping world cobalt prices relatively stable. Therefore, it would be advantageous to take price fluctuation analysis into account in stockpile management.

When predicting future prices of cobalt, one looks at short-, medium- and long-term trends in prices. This is relatively easy to do mathematically by using a least-squares fit on the previous data. Say, for instance, that you want to predict the price of cobalt n months in the future. For short-term price trends one might look at the prices for the previous 5 months. For medium-term trends one might look at the prices for the past 20 months, and in the long term for perhaps 100 months. By doing a least-squares fit to the short, medium, and long-term data, one can get different predictions on the future price, along with different error bars.

If one is looking for a pricing trend over the last m months, a least-squares linear function can be found for price as a function of time (months)

$$P_i = A + Bi$$

$$\text{Where: } i = \text{month number} \tag{9}$$

$$P_i = \text{price for that month}$$

and the parameters A and B are given by

$$A = \frac{\left(\sum_{i=-m}^{-1} i^2\right)\left(\sum_{i=-m}^{-1} P_i\right) - \left(\sum_{i=-m}^{-1} i\right)\left(\sum_{i=-m}^{-1} iP_i\right)}{\Delta}$$

$$B = \frac{m\left(\sum_{i=-m}^{-1} iP_i\right) - \left(\sum_{i=-m}^{-1} i\right)\left(\sum_{i=-m}^{-1} P_i\right)}{\Delta} \tag{10}$$

$$\Delta = m\left(\sum_{i=-m}^{-1} i^2\right) - \left(\sum_{i=-m}^{-1} i\right)^2,$$

where $i = 0$ is the present month. Thus, this gives a linear fit of price versus time for the previous m months. The error bars on the slope and intercept are given by

$$\sigma_A^2 = \frac{\varphi^2}{\Delta}\left(\sum_{i=-m}^{-1} i^2\right)$$

$$\sigma_B^2 = \frac{m\varphi^2}{\Delta} \tag{11}$$

$$\varphi^2 = \frac{1}{m-2}\left(\sum_{i=-m}^{-1} (P_i - A - Bi)^2\right).$$

This allows for a prediction of price n months in the future by

$$P_n = A + Bn,$$

with the standard deviation of error being

$$\sigma_{P_n} = \sqrt{(\sigma_A)^2 + (n\sigma_B)^2}. \tag{12}$$

The usefulness of this estimate of error in the predicted price will become apparent later.

In addition, it may be useful to predict prices based on both short- and long-term trends in prices. Different predicted prices P_j with different errors bars σ_{P_j} can be averaged together with weighting factors w_j

$$P_{\text{best}} = \frac{\sum_j w_j P_j}{\sum_j w_j} \tag{13}$$

with $w_j = 1/\sigma_j^2$ and

$$\sigma_{P_{\text{best}}} = \left(\sum_j w_j\right)^{-1/2} \tag{14}$$

Also, one can artifically weight one price prediction over another by, for example, multiplying a weighting factor w_j by 2 if one price prediction is expected to be twice as important as another.

As an example, let us take the price per pound of cobalt over the last 24 months to have been as shown in Table 1.

We predict what the price per pound of cobalt for the month of Feb 1985 ($i = 0$) should be using a short-term linear fit (the last 5 months) and a long-term fit (the last 24 months), weighting the short-term prediction (P_s) twice as much as the long-term prediction (P_l). Using the least-squares fit equations from above, we get P_s = \$5.29 ± .05 and P_l = \$4.20 ± .17. Then, by taking the average of the two, weighted by the inverse squares of their error bars and doubly weighting the short-term prediction, we get a predicted price P_{best} = \$5.23 ± .05. Thus, if the actual price per pound of cobalt in February is \$5.10 (two standard deviations below the predicted price), it is reasonable to assume that cobalt is an exceptionally good buy this month.

It should be noted that this model allows for decisions and predictions to be made on any time scale, not just monthly. A monthly time scale was used only for purposes of illustration.

Table 1. Example prices for the last 24 months.

Month	i	$\$P_i$	Month	i	$\$P_i$
March 1983	−23	$8.00	March 1984	−11	$5.70
April 1983	−22	$7.70	April 1984	−10	$5.50
May 1983	−21	$7.50	May 1984	−9	$5.40
June 1983	−20	$7.60	June 1984	−8	$5.10
July 1983	−19	$6.90	July 1984	−7	$4.80
Aug 1983	−18	$7.40	Aug 1984	−6	$4.80
Sept 1983	−17	$7.00	Sept 1984	−5	$4.70
Oct 1983	−16	$6.30	Oct 1984	−4	$4.90
Nov 1983	−15	$6.20	Nov 1984	−3	$5.00
Dec 1983	−14	$5.80	Dec 1984	−2	$5.00
Jan 1984	−13	$5.40	Jan 1985	−1	$5.20
Feb 1984	−12	$5.80	Feb 1985	0	$5.10

The ability to predict prices over the long, medium, and short term allows us to take advantage of price fluctuations in the world market. Based on the expected price of cobalt and the actual price of cobalt, we can determine whether to buy, sell, or sit tight on our stockpile. If the situation is such that we would like to buy or sell, then we need to look at how our buying or selling from the stockpile will affect the world market. This analysis is similar to that done earlier to determine the social cost of buying for the stockpile. In this case, however, we are not interested in the cost to the U.S. economy, but rather with the amount to which we can affect the world market price of cobalt. The central idea here is that we want to change the current price of cobalt from its extreme level to within the range of prices our predictions consider reasonable. If, for example, the predicted price of cobalt is $5.24 ± .05 per pound, while the actual price is $5.10 per pound (over two standard deviations away), we would like to buy enough cobalt to bring the spot-market price back within the error bounds. Thus we know the exact price difference we wish to achieve; what we need to determine is the amount of additional demand we need to create to move the world price of cobalt as we desire. Previously, we showed that

$$P = C \cdot D^{-1/\eta}. \tag{4}$$

In this case, we want to know how demand D varies with price P. Specifically, we need an expression for D if P is replaced by $k \cdot P$:

$$\begin{aligned} D &= C \cdot P^{-\eta} \\ D' &= C \cdot (kP)^{-\eta} \\ D' &= CP^{-\eta} \cdot k^{-\eta} \\ D' &= D \cdot k^{-\eta}. \end{aligned} \tag{15}$$

The amount of demand we want to create is thus:

$$D' - D = Dk^{-\eta} - D = D(k^{-\eta} - 1). \tag{16}$$

It should be noted that this created demand can actually be either positive or negative. If positive, it represents an amount which should be bought, overfilling the stockpile, simply because it is a very good price for cobalt. If negative, it represents not an artificial demand by the stockpile but rather an artificial supply, for the price is so high that money can be made by selling off a portion of the stockpiled material. One expects that this will bring the price of cobalt to more reasonable levels and that in the future the material will be bought back at a lower price.

In our example the predicted price for the month of February 1985 is \$5.24 ± .07, while the actual price is \$5.10. We wish to purchase enough cobalt on the spot market to bring the price up to \$5.19 (within our error bars), representing a price increase of 1.76 percent. Putting this into Eq. (16) gives:

$$D' - D = 55{,}000{,}000(1.0176^{0.32} - 1) = 308{,}000 \text{ pounds.} \tag{17}$$

So we need to create an artificial demand of 308,000 pounds of cobalt, which we can do by contracting for that amount on the spot market.

This is a reasonable model for buying and selling from the stockpile to the spot market, since any time the price falls two or more standard deviations outside the trend it is a relatively important fluctuation. The stabilization of prices would, in general, benefit U.S. industries. It could also be a means for the stockpile to pay for its own maintenance.

Two important points should be mentioned about this model for stockpile maintenance. First of all, it is not necessary to buy or sell from the stockpile every time the model so indicates. The beginnings of new long-term trends will initially look like fluctuations. This would be information available to the stockpile manager and unavailable to this model. Fluctuation analysis would have to wait for a while till the trend was established. Secondly, only a certain amount of buying and selling from the stockpile should be allowed, and the stockpile should never be allowed to rise or fall more than, say, ten percent from the target amount. The actual amount would be determined by the stockpile manager. However, as has been previously mentioned, a stockpile of this size (over three times U.S. annual consumption) could easily handle a certain amount of speculation in hopes of earning money and/or stabilizing prices.

DRAWING DOWN FROM A FULL STOCKPILE

The same three cases we considered in finding the rate to build the stockpile (war, supply, and no disruption) apply to the possibilities of drawing it down. In each case, the manager of the stockpile must consider what rate to decrease his holdings, any lower bound to the amount the stockpile should contain, and to whom he should sell so as to smooth over the disruption without undue risk of running out in a future disruption. The section on price fluctuations has already dealt with these questions for the last case—managing the stockpile under normal conditions.

Mitigating the first type of disruption, that due to war, is the express intent of the Strategic and Critical Materials Stockpiling Act. In such a case, the selling rate will have been determined in part by FEMA in their estimates of how much is required for a three-year war supply. The manager's job is to husband the supply so that the defense industries are not hampered, and the supply lasts for the next three years. It is to be expected that the non-essential uses of cobalt will be decreased and that the 'delayed stockpiles'—recycling, substitution, and domestic production—will begin coming on-line near the end of the three-year period. These factors will stretch the coverage of the defensive stockpile, and make it unnecessary for the manager to set lower limits to the amount he can sell during a war.

A stockpile cannot be expected to eradicate all the effects of a war on cobalt prices and availability; even if it could, it should not. There must be some incentive for developing the potential of the long-range solutions. The proper goal is to provide essential defense industries with access to this strategic metal.

The second type of disruption, that due to unstable supplying countries, is classified an economic disruption—a problem which the defensive stockpile may not attempt to alleviate according to current legislation. The Department of the Interior recognizes it as a problem which could have great cost and does have a higher probability than war[7], and we believe that we should consider dealing with it.

One of the immediate results of the crisis in Zaire in 1979 was panic buying by industries in an attempt to insure that they had enough cobalt. Producer prices increased dramatically—to $25.00 from $5.50 in 1977—and the spot market commanded prices as high as $50.00. The mere existence of a usable stockpile of cobalt would help prevent such panic buying.

Let us establish as our worst case a scenario in which Zaire, the country that produced 47 percent of the world's cobalt in 1983, cuts off all production. Consider what reserves are available to make up this shortfall: Much of Africa's mine production is refined in Belgium and elsewhere, thus the metal in transit and at refineries will provide an initial buffer. Combining this with private inventories, there is a six- to eight-month supply without a stockpile. In addition, other suppliers would be expected to increase their outputs to meet part of the demand. Altogether, the stockpile would be called on to meet less than one half of the normal domestic needs.

Notice especially that this is *normal* needs—the stockpile was intended to supply a war-time economy, with an abnormally high demand for cobalt, for three years. If the stockpile's reserves were called upon to supply half of U.S. needs for two years at 1983 rates, it would only use 17 million pounds, i.e. 20 percent of the total stockpile goal.

The stockpile manager must decide in this case how much cobalt he must keep in his stockpile in case of future disruptions and, thus, what limits to place on his selling. This is a special case of the *ruin problem*[4]: if orders of random size are placed at random intervals how much of any given order can an inventory manager fill until he gets an order which he cannot fill? In this special case, the cost of a war with a low inventory would far out-weigh the cost of a second supply disruption. Furthermore, a disruption would increase the probability of a war and could decrease the probability of a second supply disruption. Thus, for this model, we will only consider the possibility of a war following a supply disruption.

As you recall from the discussion on the social cost of war, the closer the stockpile level was to the goal, the less urgent it was to reach the goal—a 20 percent shortfall will cost less than 20 percent times the cost of war with an empty stockpile. Thus, 20 percent of the goal appears to be a reasonable limit. On the one hand, it is a high estimate for the need during a supply disruption; the Office of Mineral Policy and Research Analysis used a two year disruption as their worst-case, and the Zaire crisis was over in one[7]. On the other, it leaves a two and a half year supply at war-time consumption rates—which, with the two year disruption, would allow enough time for the 'delayed stockpiles' to come on-line and extend the coverage of the defensive stockpile.

The manager could give preference to defense industries during a supply disruption in allocating his holdings. This, however, would probably be a mistake. It would provide a disincentive for developing long-term solutions so that a war would find those industries unprepared.

STRENGTHS, WEAKNESSES, AND TESTING OF THE MODEL

Perhaps the greatest strength of the model is how well it models reality. During stockpile buildup it maximizes the benefits of early buildup while minimizing the social cost of the higher prices caused by increased demand. In the early stages of buildup, while the stockpile is still low, it calls for increased purchasing beyond the constant linear rate. Later, as the stockpile nears its target quantity, the base rate will be lower than the original rate and purchasing will slow down. This decreasing rate of purchase as the stockpile nears target is reasonable to expect from a stockpile manager.

The price-fluctuation technique is a rational way to maintain a stockpile. It makes the stockpile help pay for itself and evens out price fluctuations without significantly affecting its major purpose—maintaining a supply sufficient for a three-year conventional war. This model is realistic about the limits of what a stockpile can do. It does not attempt to solve such long-range problems as pricing trends, domestic production and recycling, and long-term supply shortages. These are problems better dealt with in other ways; a strategic stockpile is at best a short-term solution to these problems.

It should be easy to write a computer program to simulate this mathematical model. With such a program it would be simple to run the model through a variety of tests. Although this model has been tested in the examples throughout the text (the numbers in the examples closely approximate the real-life situation), the model can and should be refined with a program. Such a program could experiment with a variety of probability and 'social cost' functions. And the model is very flexible; with the computer program almost any scenario could be tested.

This model does have a problem in that it cannot easily handle the problem of long-term pricing trends. This is not a major weakness, however. With a stockpile of this size, any attempt to capitalize on long-term trends are likely to *change* those trends. It would be impossible to calculate the effects of such purchases on long-range prices.

Finally, this model is quite general. It is not limited to cobalt management. With a little work on the constants in the equations and appropriate input, it can be modified to model the management of almost any strategic stockpile.

Chromium and manganese are two examples of other materials in the defensive stockpile. Each are essential for producing steel and special-alloy steels, and neither are produced domestically—over 90 percent of U.S. consumption of each of these metals is from imports. Thus they are liable to war disruptions just as cobalt is. Further, the primary supplier for both chromium and manganese is South Africa, about which there is concern because of racial tensions[3]. With assessments of the costs and probabilities of the two types of disruptions, and economic analysis of the price elasticity of demand, our model for a strategic stockpile could be applied to these materials as well.

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